Math 101/120 Sample 1 Solutions

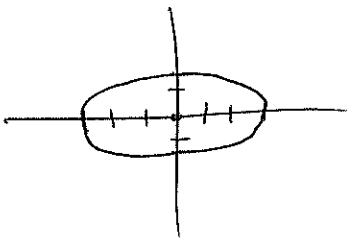
1) $(3x-2)^7$ sixth term $\binom{7}{5}(3x)^2(-2)^5$

$\frac{7!}{5!2!}(9x^2)(-32)$

$-6048x^2$

2) $\frac{4x^2}{36}+\frac{9y^2}{36}=\frac{36}{36}$

$\frac{x^2}{9}+\frac{y^2}{4}=1$



3) $2x-3y=7$

$-3y=-2x+7$

$y=\frac{2}{3}x-\frac{7}{3}$

m

perpendicular $m=-\frac{3}{2}$

$y-9=-\frac{3}{2}(x--5)$

$y-9=-\frac{3}{2}x-\frac{15}{2}+\frac{9}{1}\times\frac{2}{2}$

$y=-\frac{3}{2}x+\frac{3}{2}$

4) 3, 7, 11, ...

arithmetic $d=4$

$a_1=3$ so $a_{20}=a_1+19d$

$=3+19(4)$

$a_{20}=79$

a)

b) $S_{20}=\frac{20}{2}(3+79)$

$S_{20}=820$

5) $$\frac{\left(\frac{x-5}{x^2-4}\right)(x-2)(x+2)}{\left(\frac{x^3-3x^2-10x}{x-2}\right)(x-2)(x+2)} = \frac{x-5}{x(x^2-3x-10)(x+2)} = \frac{x-5}{x(x+2)(x-5)(x+2)}$$

$$= \frac{1}{x(x+2)(x+2)}$$

6) $$y = \frac{x+2}{x-3}$$

$$\left(x = \frac{y+2}{y-3}\right)(y-3)$$

$$xy - 3x = y + 2$$

$$xy - y = 3x + 2$$

$$y(x-1) = 3x + 2$$

$$y = \frac{3x+2}{x-1}$$

$$f^{-1}(x) = \frac{3x+2}{x-1}$$

7) $y = x^2 - 6x + 5$ parabola ∪

Vertex $\left(-\frac{b}{2a}, \right)$

$\left(-\frac{-6}{2}, \right)$

Vertex: $(3, -4)$

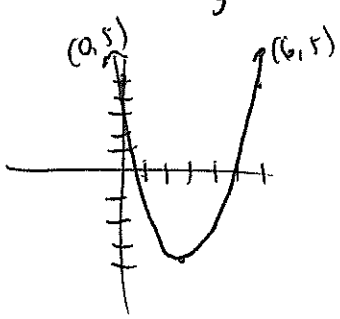
$y = 3^2 - 6(3) + 5$

$y = 9 - 18 + 5$

$y = -9 + 5$

$y = -4$

x	y
0	5
6	5



8) $\frac{x}{x+2} - \frac{7}{5-x} = \frac{14}{x^2-3x-10}$

$\left(\frac{x}{x+2} + \frac{7}{x-5} = \frac{14}{(x+2)(x-5)} \right) (x+2)(x-5)$

$x(x-5) + 7(x+2) = 14$

$x^2 - 5x + 7x + 14 = 14$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0 \quad x = -2$

check

$x = 0$ $\quad x \neq -2$

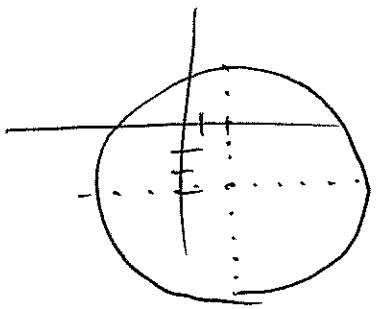
9) $x^2 - 4x + 4 + y^2 + 6y + 9 = 12 + 4 + 9$

$(x-2)^2 + (y+3)^2 = 25$

center $(+2, -3)$

radius 5

circle



10) $x + y = 30 \quad x -25$

$.25x + .40y = .30(30)$

$-25x - 25y = -750$

$25x + 40y = 900$

$15y = 150$

$y = 10$

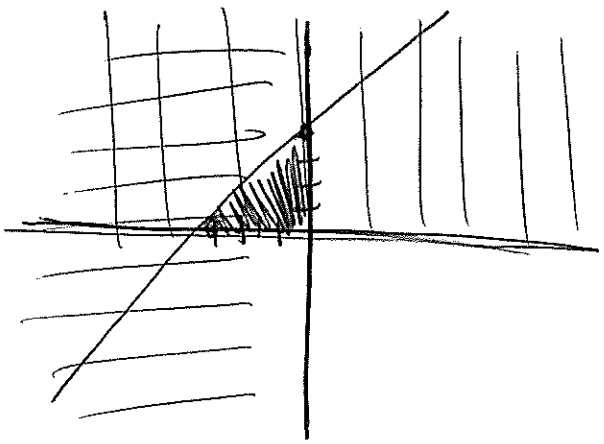
10 liters of 40%

20 liter of 25%

11) $x - y \leq -4$

$x \leq 0$

$y \geq 0$



12) $\log_2 x(x-7) = 3$

$2^3 = x(x-7)$

$0 = x^2 - 7x - 8$

$0 = (x-8)(x+1)$

$\boxed{x = 8}$ $\quad x \neq -1$

13) $\left|\frac{1}{2}x - 3\right| - \frac{7}{2} < -1\frac{2}{2}$

$+\frac{7}{2} \quad +\frac{7}{2}$

$\left|\frac{1}{2}x - 3\right| < \frac{5}{2}$

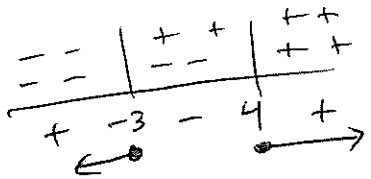
$2\left(\frac{1}{2}x - 3 < \frac{5}{2}\right) \quad \left(\frac{1}{2}x - 3 > -\frac{5}{2}\right)2$

$x - 6 < 5 \qquad x - 6 > -5$

$x < 11 \qquad x > 1$

14) $x^2 - x - 12 \geq 0$

$(x+3)(x-4) \geq 0$



15) $A = Pe^{rt}$

$3000 = 1000e^{.08t}$

$3 = e^{.08t}$

$\frac{\ln 3}{.08} = t = 13.73 \approx 14$ years

16) $(f \circ g)(x)$

$(x^3-2)^2 - 2(x^3-2) + 1$

$(x^3-2)(x^3-2)$

$x^6 - 4x^3 + 4 - 2x^3 + 4 + 1$

$\boxed{x^6 - 6x^3 + 9}$

17) $\sqrt{18-x} - 2 = x$

$(\sqrt{18-x})^2 = (x+2)^2$

$18 - x = x^2 + 4x + 4$

-18

$0 = x^2 + 5x - 14$

$0 = (x-2)(x+7)$

$x = 2 \quad x = -7$

check

$\boxed{x=2}$

$\sqrt{16} - 2 = 2$

$4 - 2 = 2$

$2 = 2$

$x = -7$

NO

18) $3x^2 - 4x + 4 = 0$

$(3x+2)(x-2) = 0$

$x = -\frac{2}{3} \quad x = 2$

NO ...

$x = \frac{-(-4) \pm \sqrt{16 - 4(3)(4)}}{2(3)}$

$= \frac{4 \pm \sqrt{16-48}}{6}$

$= \frac{4 \pm \sqrt{-32}}{6}$ $\quad -16$, 2

$= \frac{4 \pm 4i\sqrt{2}}{6} = \boxed{\frac{2 \pm 2i\sqrt{2}}{3}}$

19) $\frac{f(x+h) - f(x)}{h}$

$\frac{2(x+h)^2 + 3(x+h) - 4 - (2x^2 + 3x - 4)}{h}$

$\frac{2x^2 + 4xh + 2h^2 + 3x + 3h - 4 - 2x^2 - 3x + 4}{h}$

$\boxed{4x + 2h + 3}$

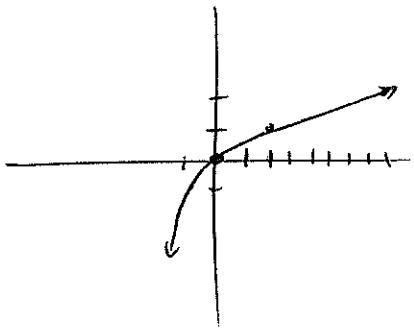
$\frac{f(x) - f(a)}{x - a}$

$\frac{(2x^2 + 3x - 4) - (2a^2 + 3a - 4)}{x - a}$

$\frac{2(x^2 - a^2) + 3(x - a)}{x - a} = \frac{(x-a)[2(x+a)+3]}{x-a} = \boxed{2x + 2a + 3}$

20) $y = \log_3(x+1)$ $\quad 3^y = x+1$

x	y
0	0
2	1
8	2
$-\frac{2}{3}$	-1



21) \$20,000

Year 1 = $\left|20{,}000 \times \frac{1}{6} - 20{,}000\right| = \left|3{,}333.33 \times \frac{1}{6} - 16666.67\right|$

Year 2

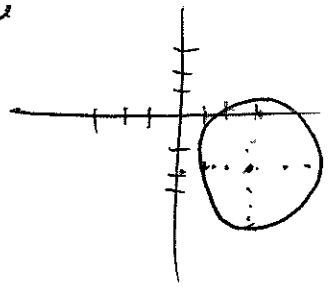
Year 3 = $\left|13{,}888.89 \times \frac{1}{6} - \quad\right|$

Year 4 = $\left|11\,574.07 \times \frac{1}{6} - \quad\right|$

Year 5 = \$9,645.06

22) $\dfrac{16^{-1/2} x^{-6/2} y^{+4/2}}{(-27)^{1/3} x^{-6/3} y^{3/3}} = \dfrac{x^2 y^2}{4(-3) x^3 y^1} = \dfrac{y}{-12x}$

23) $(x-3)^2 + (y+2)^2 = 4$ circle



a) center $(+3, -2)$

radius 2

b) Domain $1 \le x \le 5$

Range $0 \le y \le -4$

c) Not a function

24) $(0, 1, -1)$