**Lab: Law of Sines/Cosines Lab: *(Instructor Version)***

*This lab is a discovery lab and should be done prior to presenting the formulas during lecture.*

**Motivation:** To not just see and memorize formulas, but understand the geometry supporting the Law of Sines and the Law of Cosines.

**Objectives:** Students will be able to verify the Law of Sines and derive the Law of Cosines.

**Materials Needed:**

Protractor, Ruler, Paper,Pencil

**Activity 1: Law of Sines**

*This activity should take 20-30 minutes. Your students will use the straight edge of their protractor to draw a triangle and answer the following questions. ( All triangles should be different so that students can see that the Law of Sines is true for all triangles.)*

1. What are the measures of each of the angles?
2. What are the lengths of each of the sides?
3. Calculate the following ratios: $\frac{\sin(A)}{a}$, $\frac{\sin(B)}{b}$, and $\frac{\sin(C)}{c}$.
4. Based on your calculations in 3., can you draw any conclusions? *(Think, pair, share is a good activity to use for this question.)*
5. Will your conclusions work with any triangle? Why or why not? *(Think, pair, share is a good activity to use for this question.)*

**Activity 2: Law of Cosines**

*This activity should take about 30-40 minutes.*

*You should lead your students through the drawing of the triangle and inform your students that their triangles should look different than the person next to them.*

Draw triangle $∆ABC$. From vertex $C$, draw an altitude (height of the triangle) of length $k$. Separate side $c$ into segments $x$ and $c-x$. *(Question you can pose to your class: Why can the segments be represented in this way?)*

*Students will follow the steps below to derive the Law of Cosines. Good steps to use the think, pair, share activity will have a TPS label on them.*

1. The altitude separates $∆ABC$ into two right triangles. What are the triangles?
2. *TPS:* Use the Pythagorean Theorem to write two equations, one relating $k, b,$ and $c-x$, and another relating $a, k,$ and $x$.
3. *TPS:* Notice that both equations contain $k^{2}$. Why is this so?
4. Solve each equation for $k^{2}$.
5. Since both of the equations in Question 4 are equal to $k^{2}$, we can set them equal to each other. *(Question you can pose to the class: Why is this true?)*
6. Set the equation equal to each other to form a new equation. But notice now that the equation you just formed involves $x$. However, $x$ is not a side of $∆ABC$. As a result we will attempt to rewrite the newly found equation so that it does not include $x$. Begin by expanding the equation and combining like terms then solve the equation for $b^{2}$.
7. The resulting equation in Question 6 still includes $x$. To eliminate $x$ from the equation we need to substitute an equivalent expressions for $x$. Try finding a relationship between $\cos(B)$ and $x$ that we can substitute into the equation. *(Question to pose to your students:* *Why should we use* $\cos(B)$*?)*
8. Use the relationship from Question 6 and solve for $x$. *(Question to pose to your students: Why should we solve for* $x$*?)*
9. Substitute the equivalent expression for $x$ into the equation from Question 6. This equation is called the **Law of Cosines**.
10. Using this same method, two more equations could be written for $a^{2}$ and $c^{2}$. Without going through the whole process again, write two other forms of the law of cosines for $∆ABC$.

**Part 3:**

*This activity would be great using the think, pair, share technique and should take about 10-15 minutes.*

1. What could the Law of Sines and the Law of Cosines be used for?
2. Can you think of a type of problem where the Law of Sines would be used over the Law of Cosines and vice versa.