**Lab: Law of Sines/Cosines Lab:**

**Motivation:** To not just see and memorize formulas, but understand the geometry supporting the Law of Sines and the Law of Cosines.

**Objectives:** Students will be able to verify the Law of Sines and derive the Law of Cosines.

**Materials Needed:**

Protractor, Ruler, Paper,Pencil

**Activity 1: Law of Sines**

Use the straight edge of their protractor to draw a triangle and answer the following questions.

1. What are the measures of each of the angles?
2. What are the lengths of each of the sides?
3. Calculate the following ratios: $\frac{\sin(A)}{a}$, $\frac{\sin(B)}{b}$, and $\frac{\sin(C)}{c}$.
4. Based on your calculations in 3., can you draw any conclusions?
5. Will your conclusions work with any triangle? Why or why not?

**Activity 2: Law of Cosines**

Follow the steps below to derive the Law of Cosines formula:

1. Draw a triangle $∆ABC$. From vertex $C$, draw an altitude (height of the triangle) of length $k$. Separate side $c$ into segments $x$ and $c-x$.

1. Why can the segments be represented in this way?
2. The altitude separates $∆ABC$ into two right triangles. What are the triangles?
3. Use the Pythagorean Theorem to write two equations, one relating $k, b,$ and $c-x$, and another relating $a, k,$ and $x$.
4. Notice that both equations contain $k^{2}$. Why is this so?
5. Solve each equation for $k^{2}$.
6. Since both of the equations in Question 4 are equal to $k^{2}$, we can set them equal to each other. Why is this true?
7. Set the equations equal to each other to form a new equation.
8. But notice now that the equation you just formed involves $x$. However, $x$ is not a side of $∆ABC$. As a result we will attempt to rewrite the newly found equation so that it does not include $x$. Begin by expanding the equation and combining like terms then solve the equation for $b^{2}$.
9. The resulting equation in Question 6 still includes $x$. To eliminate $x$ from the equation we need to substitute an equivalent expressions for $x$. Try finding a relationship between $\cos(B)$ and $x$ that we can substitute into the equation. Why should we use $\cos(B)$?
10. Use the relationship from Question 6 and solve for $x$. Why should we solve for $x$?
11. Substitute the equivalent expression for $x$ into the equation from Question 6. This equation is called the **Law of Cosines**.
12. Using this same method, two more equations could be written for $a^{2}$ and $c^{2}$. Without going through the whole process again, write two other forms of the law of cosines for $∆ABC$.

**Activity 3: Follow up questions:**

1. What could the Law of Sines and the Law of Cosines be used for?
2. Can you think of a type of problem where the Law of Sines would be used over the Law of Cosines and vice versa.