

1. Jan makes a 10% down payment on a used car and pays off the balance in quarterly payments of \$540 for one year. Find the cost of the car.
- A. \$2400    B. \$2500    C. \$2700    D. \$2750    E. \$3000
2. The lines with equations  $y = mx + 1$  and  $y = 1 - mx$ ,  $m \neq 0$ , intersect at  $(r, s)$ . Find the value of  $r + s$ .
- A. -2    B. -1    C. 0    D. 1    E. 2
3. A town has 3.75 people per family and 2.5 TV's per family. Find the number of TV's per person.
- A.  $\frac{3}{5}$     B.  $\frac{2}{3}$     C.  $\frac{3}{4}$     D.  $\frac{4}{5}$     E.  $\frac{5}{6}$
4. Which of the following five numbers is the mean (average) of the other four numbers?
- A. 37    B. 49    C. 51    D. 53    E. 75
5. The sum of the slopes of the perpendicular lines with equations  $y = mx$  and  $y = Mx$  equals  $\frac{9}{20}$ . Find  $|M - m|$ .
- A.  $\frac{29}{20}$     B.  $\frac{33}{20}$     C.  $\frac{37}{20}$     D.  $\frac{41}{20}$     E.  $\frac{43}{20}$
6. The only integer pair with a difference equal to the quotient is  $(4, 2)$ . Find the sum of the only pair of positive fractions with denominator 4 in lowest terms with this property.
- A. 6.5    B. 7    C. 7.5    D. 8    E. 8.5
7. A print shop takes 2.4 hr to run a job on the X10 and X5 copiers. The same job takes 3 hr on the X10 and X2 copiers, and 4 hr on the X5 and X2 copiers. How many hours would the job take running on all three machines?
- A. 1.2    B. 1.5    C. 1.6    D. 1.8    E. 2.0
8. In the equation  $AMA + TYC = AWAY$ , identical letters are replaced by identical digits and different letters are replaced by different digits 0-9 to produce one 4-digit and two 3-digit numbers. If leading digits cannot be 0, how many different values of Y are possible?
- A. 3    B. 4    C. 5    D. 6    E. 7
9. How many irrational solutions does the equation  $18x^4 - 11x^2 + 1 = 0$  have?
- A. 0    B. 1    C. 2    D. 3    E. 4
10. You win a certain game by either pushing a button once and getting a green light, or pushing the button twice and getting either 2 green lights or exactly one red light. The probability  $p > 0$  of a green light on a single push is constant and three times the probability of a red light. Find the value of  $p$  for which the probability of winning is the same for either the one-push or two-push option.
- A.  $\frac{3}{8}$     B.  $\frac{3}{7}$     C.  $\frac{2}{5}$     D.  $\frac{1}{2}$     E.  $\frac{3}{5}$
11. For the function  $f(x)$ ,  $f(1) = 3$ . Also,  $f(x) + f(y) = f\left(\frac{x+y}{2}\right)f\left(\frac{x-y}{2}\right)$  for all real numbers  $x$  and  $y$ . Write the value of  $f(6)$  in the corresponding blank on the answer sheet.

12. Find  $(\sin 15^\circ)(2 \cos 15^\circ)(2 \cos 30^\circ)(2 \cos 60^\circ)\dots(2 \cos 960^\circ)(2 \cos 1920^\circ)$ .

- A.  $-\frac{\sqrt{3}}{2}$  B.  $-\frac{1}{2}$  C.  $\frac{1}{2}$  D.  $\frac{\sqrt{3}}{2}$  E. 1

13. The equation  $a^6 + b^2 + c^2 = 2014$  has only one solution in positive integers for which  $b$  is prime. Find  $a + b + c$ . A. 50 B. 52 C. 54 D. 56 E. 58

14. A closed curve is a curve that begins and ends at the same point. A closed curve which passes through point  $P$  twice divides the plane into 3 regions. If a closed curve passes through each of the 20 points  $P_1, P_2, \dots, P_{20}$  exactly twice and each of the 12 points  $P_{21}, P_{22}, \dots, P_{32}$  exactly three times (and passes through no other points multiple times), into how many regions does the curve divide the plane?

- A. 32 B. 34 C. 44 D. 46 E. 48

15. A domino is a  $1 \times 2$  rectangle. Find the number of different ways 8 identical dominos can be arranged to form a  $4 \times 4$  square (assume that arrangements which are rotations or reflections of each other are different). A. 24 B. 32 C. 36 D. 40 E. 48

16. To determine divisibility by 7, subtract twice the last digit from the number formed by removing the last digit; the result is divisible by 7 exactly when the original number is. If you subtract  $k$  times the last digit instead of twice the last digit, for what value of  $k$  does this determine divisibility by 13?

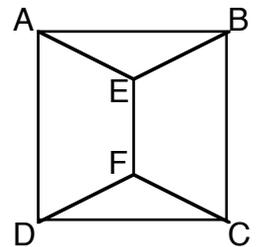
- A. 5 B. 6 C. 7 D. 8 E. 9

17. Given the points  $A(0, 0)$ ,  $B(3, 9)$ ,  $C(9, 3)$ , and  $D(6, 6)$ , find the area of the region in the first quadrant consisting of those points closer to  $A$  than to any of the other 3 points.

- A. 14.5 B. 15.25 C. 16.25 D. 16.5 E. 18.75

18. Square  $ABCD$ , shown at the right, has  $AB = 6$ . If  $AE = BE = EF = DF = CF$ , find  $EF$ .

- A.  $\frac{6 + \sqrt{15}}{3}$  B.  $2\sqrt{7} - 2$  C.  $\frac{79}{24}$  D.  $2(\sqrt{2} + \sqrt{3}) - 3$  E.  $\frac{8 - \sqrt{2}}{2}$



19. The sequence  $\{a_n\}$  satisfies  $a_n = a_{n-1} + a_{n-3}$  for all  $n \geq 4$ . If  $a_1 = 3$  and  $a_6 = 30$ , find  $a_8$ .

- A. 63 B. 66 C. 69 D. 72 E. 75

20. For rational numbers  $A, B, C$ , let  $P(x) = Ax^5 + Bx^4 + Cx^3 + Cx^2 + Bx + A$ . If  $\sqrt{3} - 1$  is one solution of the equation  $P(x) = 0$ , which of the following is also a solution of  $P(x) = 0$ ?

- A.  $1 - \sqrt{3}$  B.  $\frac{1 + \sqrt{3}}{2}$  C.  $-1$  D. both A and C E. both B and C

Test #1    **AMATYC Student Mathematics League**    October/November 2014

1. A
2. D
3. B
4. D
5. D
6. C
7. E
8. A
9. C
10. B
11. 322
12. A
13. B
14. D
15. C
16. E
17. D
18. B
19. A
20. E