1. Let p be the price of the car. After paying 10%, Jan owes $0.90p$. If $0.90p/4 = 540$ it follows that $p = 2400$. (Answer: A)

2. Both lines have a y-intercept at $(0,1)$. $0 + 1 = 1$. Yeah, it's that simple. Hopefully you didn't skip this one. (Answer: D)

3. $\dfrac{P}{F} = 3.75$ and $\dfrac{T}{F} = 2.5$. It follows that $\dfrac{T}{P} = \dfrac{2.5}{3.75} = \dfrac{2}{3}$. (Answer: B)

4. Guess and check. $53 = \dfrac{37 + 49 + 51 + 75}{4}$. (Answer: D)

5. We know $M = -\frac{1}{m}$. So $m - \frac{1}{m} = \frac{9}{20} \implies m^2 - \frac{9}{20}m - 1 = 0 \implies m = \frac{5}{4}, -\frac{4}{5} \implies |M - m| = \frac{41}{40}$. (Answer: D)

6. The numbers are in the form $\frac{x}{4}$ and $\frac{y}{4}$ where x and y are integers that are not divisible by 2 or 4. $\frac{x}{4} - \frac{y}{4} = \frac{x}{y} \implies x = \frac{y^2}{y-4}$. Use the TABLE function or trial and error to find $x = 25$ and $y = 5$. $\frac{25}{4} + \frac{5}{4} = 7.5$. (Answer: C)

7. Let x be the rate of the X2 copier, y be the rate of the X5 and z be the rate of the X10. We get the following system of equations: $2.5y + 2.5z = 1$, $3x + 3z = 1$ and $4x + 4y = 1$. Divide each equation by its corresponding rate to get: $y + z = \frac{5}{12}$, $x + z = \frac{1}{3}$ and $x + y = \frac{1}{4}$. Add all three equations together to get $2x + 2y + 2z = 1$ which represents all three machines completing the job in 2 hours. (Answer: E)

8. The sum of two 3-digit numbers must be less than 2000, so A must be 1. Looking at the second digits, neither M nor Y can be 1, so M + Y can't be 1 so it must be 11. Now looking at the hundreds digit, the 1 will carry so 1 + 1 + T must be greater than or equal to 10, which means T is either 8 or 9. But if T is 9, W would be 1 which is already taken so T must be 8 and W must be 0. We now have 1M1 + 8YC = 101Y with M + Y = 11 and 1 + C = Y. Use trial and error to find C can only be 3, 4, or 6. (Answer: A)

9. Factor to get $(9x^2 - 1)(2x^2 - 1) = 0 \implies x = \pm\frac{1}{3}, \pm\frac{1}{\sqrt{2}}$. (Answer: C)

10. $p = p^2 + 2\left(1 - \dfrac{p}{3}\right)\left(\dfrac{p}{3}\right) \implies \dfrac{p}{3}\left(\dfrac{7p}{3} - 1\right) = 0$. (Answer: B)

11. Let $f(2) = a$. $f(1) + f(1) = f(1)f(0) \implies f(0) = 2$. $f(2) + f(0) = f(1)f(1) \implies f(2) = 7$. $f(3) + f(1) = f(2)f(1) \implies f(3) = 18$. $f(6) + f(0) = f(3)f(3) \implies f(6) = 322$. (Answer: 322)

12. Applying the double angle identity to $2\sin 15^\circ \cos 15^\circ$ – you get $\sin 30^\circ$. Now apply the double angle identity to $2\sin 30^\circ \cos 30^\circ$ and you get $\sin 60^\circ$. Repeat this process until the entire expression collapses to get $\sin 3840^\circ$. Now, $3840^\circ = 10 \times 360^\circ + 240^\circ$ so $\sin 3840^\circ = \sin 240^\circ = -\sqrt{3}/2$.(Answer: A)

13. Evaluating $\sqrt[6]{2014}$ on your calculator will give you the maximum possible value for a, which is 3. So a can be only 1, 2 or 3. Try $a = 3$. Using the TABLE function, quickly scan for integer values of $Y = \sqrt{2014 - 3^6 - X^2}$. You'll find $(14, 33)$ but neither is prime. Continue down the table to find $(18, 31)$ so $3^6 + 31^2 + 18^2 = 2014$. (Answer: B)

14. Look at a smaller sample. Let the closed curve pass through the points P_1, P_2, P_3 twice and through the points P_4 and P_5 three times. You can see that for each of the first three points, you have one closed area, for each of the last two points you have 2 closed areas, plus the inside of the curve and the outside of the curve. Therefore, for 20 points and 12 points, you will have $1 \times 20 + 2 \times 12 + 2 = 36$. (Answer: D)

15. Consider the number of ways to arrange 4 dominos on the upper half of the grid. There are 5 such ways shown in *Figure 1*. For each of the 5 arrangements on the top 8 squares, you have 5 similar arrangements on the bottom 8 squares, which makes $5 \times 5 = 25$ ways.

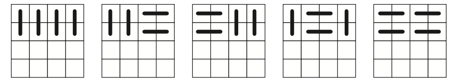


Figure 1

Now consider the case when the left 8 squares are arranged like in *Figure 2*. With the remaining 8 squares on the right side, there are 5 ways to arrange the dominos. Similarly, if we arrange the right 8 squares as shown in *Figure 2*, we will get 5 more arrangements which brings the total to 35. Finally, *Figure 3* shows the 36th unique arrangement. (Answer: C)

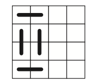


Figure 2

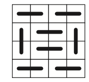
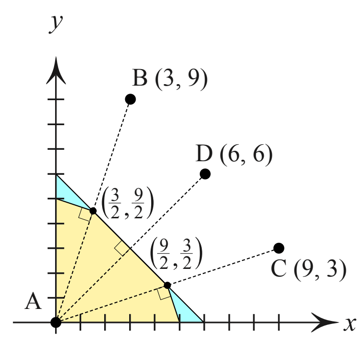


Figure 3

16. Take 13, remove the 3 and subtract $3k$ and you get $1 - 3k$. Test each of the answers given and you'll find $1 - 3(9) = -26$ is the first one divisible by 13. You can also try $2 - 6k$ (for 26) and $3 - 9k$ (for 39) and each of those are divisible by 13 when $k = 9$. Note, this can be done pretty quickly using the TABLE function on your calculator. (Answer: E)

17. The set of points that are equidistant from two given points is the perpendicular bisector of the line segment connecting the two points. If we draw the perpendicular bisector of the line segment between A $(0, 0)$ and D $(6, 6)$ it forms a 45°-45°-90° triangle with sides of length 6. The area of this triangle is $\frac{1}{2}(6)(6) = 18$. This represents all of the points that are closer to A than to D. Now draw in the perpendicular bisector of $\overline{\text{AB}}$ and $\overline{\text{AC}}$. This removes two congruent triangles that have height 3/2 and base 1, each with an area of 3/4. $18 - 2(3/4) = 16.5$. (Answer: D)



18. Consider trapezoid AEFD with altitude $h = 3$. Let AE $= x$. AD can be expressed as the sum of EF and 2 times the short leg of the right triangle formed by drawing an altitude from E to side AD. This gives: $6 = 2\sqrt{x^2 - 9} + x$ (Answer: B)

19. $a_4 = a_3 + 3$, $a_5 = a_4 + a_2 = a_3 + a_2 + 3$, $a_6 = 30 = a_5 + a_3 = 2a_3 + a_2 + 3 \implies 2a_3 + a_2 = 27$. $a_7 = 30 + a_4$. $a_8 = a_7 + a_5 = 36 + 2a_3 + a_2$. We know from the equation for a_6 that $2a_3 + a_2 = 27$, $\therefore a_8 = 36 + 27$. (Answer: A)

20. You can quickly plug in $x = -1$ and verify it's a root. The question now becomes: Is $x = 1 - \sqrt{3}$ or is $x = (1 + \sqrt{3})/2$ a root? If $x = \sqrt{3} - 1$ is a root, then $x = -\sqrt{3} - 1$ (its conjugate) is also a root. So we can write: $P(x) = A(x - p_1)(x - p_2)(x + 1)(x - (-1 + \sqrt{3}))(x - (-1 + \sqrt{3}))$ where p_1 and p_2 are the remaining roots. Based on the fact that the leading term of P is Ax^5 and the constant term is A, we can conclude that the product of all of the roots must be 1. Furthermore, if either $x = 1 - \sqrt{3}$ or $x = (1 + \sqrt{3})/2$ is a root, then their conjugates are roots. Testing each one we find that $\left(\frac{1 + \sqrt{3}}{2}\right) \cdot \left(\frac{1 - \sqrt{3}}{2}\right) \cdot (1) \cdot (-1 + \sqrt{3}) \cdot (-1 - \sqrt{3}) = 1$. (Answer: E)