Section 2.2 Velocity
2, 5, 6, 9, **12, 16**, 17, 18

Problem #2

2. Light travels at a speed of about 3×10^8 m/s. How many miles does a pulse of light travel in a time interval of 0.1 s, which is about the blink of an eye? Compare this distance to the diameter of Earth.

Solution:

At constant speed, $c = 3\times10^8$ m/s, the distance light travels in 0.1 s is

$$\Delta x = c(\Delta t) = (3\times10^8\ \text{m/s})(0.1\ \text{s}) = (3\times10^7\ \text{m})\left(\frac{1\ \text{mi}}{1.609\ \text{km}}\right)\left(\frac{1\ \text{km}}{10^3\ \text{m}}\right) = \boxed{2\times10^4\ \text{mi}}$$

Comparing this to the diameter of the Earth, D_E, we find

$$\frac{\Delta x}{D_E} = \frac{\Delta x}{2R_E} = \frac{3.0\times10^7\ \text{m}}{2(6.38\times10^6\ \text{m})} = \boxed{2.4} \quad (\text{with } R_E = \text{Earth's radius})$$

Problem #5

Two boats start together and race across a 60-km-wide lake and back. Boat A goes across at 60 km/h and returns at 60 km/h. Boat B goes across at 30 km/h, and its crew, realizing how far behind it is getting, returns at 90 km/h. Turnaround times are negligible, and the boat that completes the round trip first wins. (a) Which boat wins and by how much? (Or is it a tie?) (b) What is the average velocity of the winning boat?

Solution:

(a) Boat A requires 1.0 h to cross the lake and 1.0 h to return, total time 2.0 h. Boat B requires 2.0 h to cross the lake at which time the race is over.

$\boxed{\text{Boat A wins, being 60 km ahead of B}}$ when the race ends.

(b) Average velocity is the net displacement of the boat divided by the total elapsed time. The winning boat is back where it started, its displacement thus being zero, yielding an average velocity of $\boxed{\text{zero}}$.

Problem #6

A graph of position versus time for a certain particle moving along the x-axis is shown in Figure P2.6. Find the

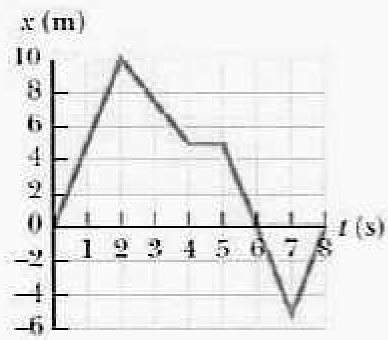


FIGURE P2.6 (Problems 6 and 17)

average velocity in the time intervals from (a) 0 to 2.00 s, (b) 0 to 4.00 s, (c) 2.00 s to 4.00 s, (d) 4.00 s to 7.00 s, and (e) 0 to 8.00 s.

Solution:

The average velocity over any time interval is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

(a) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{10.0\text{ m} - 0}{2.00\text{ s} - 0} = \boxed{5.00\text{ m/s}}$$

(b) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{5.00\text{ m} - 0}{4.00\text{ s} - 0} = \boxed{1.25\text{ m/s}}$$

(c) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{5.00\text{ m} - 10.0\text{ m}}{4.00\text{ s} - 2.00\text{ s}} = \boxed{-2.50\text{ m/s}}$$

(d) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-5.00\text{ m} - 5.00\text{ m}}{7.00\text{ s} - 4.00\text{ s}} = \boxed{-3.33\text{ m/s}}$$

(e) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0 - 0}{8.00\text{ s} - 0} = \boxed{0}$$

Problem # 9

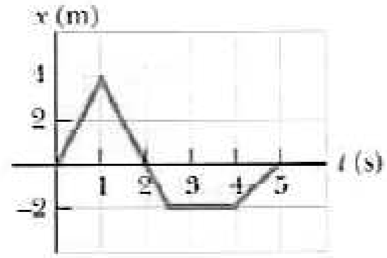


FIGURE P2.8 (Problems 8 and 9)

Find the instantaneous velocities of the tennis player of Figure P2.8 at (a) 0.50 s, (b) 2.0 s, (c) 3.0 s, and (d) 4.5 s.

<u>Solution</u>:

The instantaneous velocity at any time is the slope of the x vs. t graph at that time. We compute this slope by using two points on a straight segment of the curve, one point on each side of the point of interest.

(a) $$v|_{0.50\text{ s}} = \frac{x|_{1.0\text{ s}} - x|_{t=0}}{1.0\text{ s} - 0} = \frac{4.0\text{ m}}{1.0\text{ s}} = \boxed{4.0\text{ m/s}}$$

(b) $$v|_{2.0\text{ s}} = \frac{x|_{2.5\text{ s}} - x|_{1.0\text{ s}}}{2.5\text{ s} - 1.0\text{ s}} = \frac{-6.0\text{ m}}{1.5\text{ s}} = \boxed{-4.0\text{ m/s}}$$

(c) $$v|_{3.0\text{ s}} = \frac{x|_{4.0\text{ s}} - x|_{2.5\text{ s}}}{4.0\text{ s} - 2.5\text{ s}} = \frac{0}{1.5\text{ s}} = \boxed{0}$$

(d) $$v|_{4.5\text{ s}} = \frac{x|_{5.0\text{ s}} - x|_{4.0\text{ s}}}{5.0\text{ s} - 4.0\text{ s}} = \frac{+2.0\text{ m}}{1.0\text{ s}} = \boxed{2.0\text{ m/s}}$$

Problem # 12

An athlete swims the length L of a pool in a time t_1 and makes the return trip to the starting position in a time t_2. If she is swimming initially in the positive x-direction, determine her average velocities symbolically in (a) the first half of the swim, (b) the second half of the swim, and (c) the round trip. (d) What is her average speed for the round trip?

Solution:

(a) $$\bar{v}_1 = \frac{(\Delta x)_1}{(\Delta t)_1} = \frac{+L}{t_1} = \boxed{+L/t_1}$$

(b) $$\bar{v}_2 = \frac{(\Delta x)_2}{(\Delta t)_2} = \frac{-L}{t_2} = \boxed{-L/t_2}$$

(c) $$\bar{v}_{\text{total}} = \frac{(\Delta x)_{\text{total}}}{(\Delta t)_{\text{total}}} = \frac{(\Delta x)_1 + (\Delta x)_2}{t_1 + t_2} = \frac{+L - L}{t_1 + t_2} = \frac{0}{t_1 + t_2} = \boxed{0}$$

(d) $$(\text{ave. speed})_{\text{trip}} = \frac{\text{total distance traveled}}{(\Delta t)_{\text{total}}} = \frac{|(\Delta x)_1| + |(\Delta x)_2|}{t_1 + t_2} = \frac{|+L| + |-L|}{t_1 + t_2} = \boxed{\frac{2L}{t_1 + t_2}}$$

Problem # 16

One athlete in a race running on a long, straight track with a constant speed v_1 is a distance d behind a second athlete running with a constant speed v_2. (a) Under what circumstances is the first athlete able to overtake the second athlete? (b) Find the time t it takes the first athlete to overtake the second athlete, in terms of d, v_1, and v_2. (c) At what minimum distance d_2 from the leading athlete must the finish line be located so that the trailing athlete can at least tie for first place? Express d_2 in terms of d, v_1, and v_2 by using the result of part (b).

Solution:

(a) In order for the trailing athlete to be able to catch the leader, his speed (v_1) must be greater than that of the leading athlete (v_2), and the distance between the leading athlete and the finish line must be great enough to give the trailing athlete sufficient time to make up the deficient distance, d.

(b) During a time t the leading athlete will travel a distance $d_2 = v_2 t$ and the trailing athlete will travel a distance $d_1 = v_1 t$. Only when $d_1 = d_2 + d$ (where d is the initial distance the trailing athlete was behind the leader) will the trailing athlete have caught the leader. Requiring that this condition be satisfied gives the elapsed time required for the second athlete to overtake the first:

$$d_1 = d_2 + d \quad \text{or} \quad v_1 t = v_2 t + d$$

giving

$$v_1 t - v_2 t = d \quad \text{or} \quad \boxed{t = \frac{d}{(v_1 - v_2)}}$$

(c) In order for the trailing athlete to be able to at least tie for first place, the initial distance D between the leader and the finish line must be greater than or equal to the distance the leader can travel in the time t calculated above (i.e., the time required to overtake the leader). That is, we must require that

$$D \geq d_2 = v_2 t = v_2 \left[\frac{d}{(v_1 - v_2)} \right] \quad \text{or} \quad \boxed{D \geq \frac{v_2 d}{v_1 - v_2}}$$

Problem # 17

A graph of position versus time for a certain particle moving along the x-axis is shown in Figure P2.6. Find the instantaneous velocity at the instants (a) $t = 1.00$ s, (b) $t = 3.00$ s, (c) $t = 4.50$ s, and (d) $t = 7.50$ s.

Solution:

The instantaneous velocity at any time is the slope of the x vs. t graph at that time. We compute this slope by using two points on a straight segment of the curve, one point on each side of the point of interest.

(a) $$v_{t=1.00\text{ s}} = \frac{10.0\text{ m} - 0}{2.00\text{ s} - 0} = \boxed{5.00\text{ m/s}}$$

(b) $$v_{t=3.00\text{ s}} = \frac{(5.00 - 10.0)\text{ m}}{(4.00 \quad 2.00)\text{ s}} = \boxed{-2.50\text{ m/s}}$$

(c) $$v_{t=4.50\text{ s}} = \frac{(5.00 - 5.00)\text{ m}}{(5.00 - 4.00)\text{ s}} = \boxed{0}$$

(d) $$v_{t=7.50\text{ s}} = \frac{0 \quad (\ 5.00\text{ m})}{(8.00 - 7.00)\text{ s}} = \boxed{5.00\text{ m/s}}$$

Problem # 18

A race car moves such that its position fits the relationship

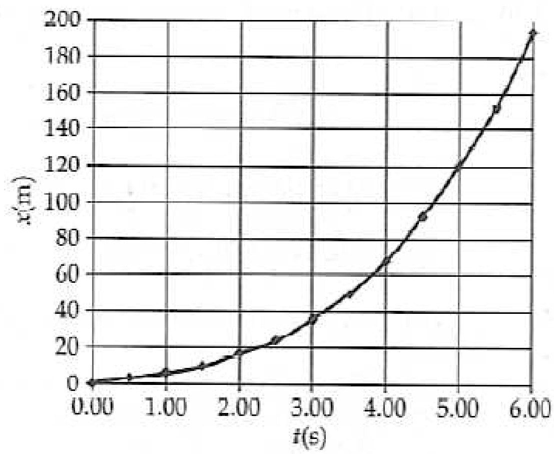
$$x = (5.0 \text{ m/s})t + (0.75 \text{ m/s}^3)t^3$$

where x is measured in meters and t in seconds. (a) Plot a graph of the car's position versus time. (b) Determine the instantaneous velocity of the car at $t = 4.0$ s, using time intervals of 0.40 s, 0.20 s, and 0.10 s. (c) Compare the average velocity during the first 4.0 s with the results of part (b).

Solution:

(a) A few typical values are

t(s)	x(m)
1.00	5.75
2.00	16.0
3.00	35.3
4.00	68.0
5.00	119
6.00	192



(b) We will use a 0.400 s interval centered at $t = 4.00$ s. We find at $t = 3.80$ s, $x = 60.2$ m and at $t = 4.20$ s, $x = 76.6$ m. Therefore,

$$v = \frac{\Delta x}{\Delta t} = \frac{16.4 \text{ m}}{0.400 \text{ s}} = \boxed{41.0 \text{ m/s}}$$

Using a time interval of 0.200 s, we find the corresponding values to be: at $t = 3.90$ s, $x = 64.0$ m and at $t = 4.10$ s, $x = 72.2$ m. Thus,

$$v = \frac{\Delta x}{\Delta t} = \frac{8.20 \text{ m}}{0.200 \text{ s}} = \boxed{41.0 \text{ m/s}}$$

For a time interval of 0.100 s, the values are: at $t = 3.95$ s, $x = 66.0$ m, and at $t = 4.05$ s, $x = 70.1$ m. Therefore,

$$v = \frac{\Delta x}{\Delta t} = \frac{4.10 \text{ m}}{0.100 \text{ s}} = \boxed{41.0 \text{ m/s}}$$

(c) At $t = 4.00$ s, $x = 68.0$ m. Thus, for the first 4.00 s,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{68.0 \text{ m} - 0}{4.00 \text{ s} - 0} = \boxed{17.0 \text{ m/s}}$$

This value is $\boxed{\text{much less}}$ than the instantaneous velocity at $t = 4.00$ s.